Review

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Topological photonics: Where do we go from here?

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Abstract: Topological photonics is currently one of the most active research areas in optics and also one of the spearheads of research in topological physics at large. We are now more than a decade after it started. Topological photonics has already proved itself as an excellent platform for experimenting with concepts imported from condensed matter physics. But more importantly, topological photonics has also triggered new fundamental ideas of its own and has offered exciting applications that could become real technologies in the near future.

Keywords: lasers; photonics; topological insulators; topological photonics.

1 Introduction

Topological photonics has started off in a proposal suggesting how to emulate the concepts of topological insulators in an electromagnetic (EM) system [1]. At that time, these concepts were strictly within the realm of condensed matter system. To understand the underlying principles, it is instructive to briefly review the basics of topological insulators. We shall do it on an intuitive basis and attempt to view these principles through the physically observable quantities. In a single sentence – topological insulators are materials that are insulators in the bulk but are perfect conductors on their edges. The robust conduction on the edge is manifested in the fact that the current there is lossless even in the presence of disorder or defects and does not depend on the shape of the edge. That is – the current continues to flow without being scattered into the bulk or being backscattered by local defects, by disorder in the lattice, or by sharp edges. Due to the lack of backscattering, such topological edge current is often viewed as unidirectional. This property is often called "topologically protected transport", and it is illustrated in Figure 1a. It is this property that made topological insulators so important – because – apart from the fundamental physics involved – having a mechanism that can bring to lossless flow of energy, charges, or information, is extremely important for any applications. The important parameter determining this unique robustness is the strength of the variations of the potential that would have normally caused scattering, not the shape nor the position of a particular defect or disorder in the lattice.

To understand the essence of topological transport, it is instructive to recall the first topological insulator ever discovered: the integer quantum Hall effect [2]. This phenomenon occurs in semiconductors under low temperature and a strong magnetic field (Figure 1b). The Lorentz force opens a bandgap in the dispersion curve, and the edge states are characterized by a single line crossing the gap in diagonal. This oversimplified picture of the quantum Hall effect is sketched in Figure 1c, where the red line marks the dispersion curve of the edge states. Notice that there is only one line, as there is no line crossing the gap in the opposite direction. The slope of this line provides the group velocity of any edge excitation (superposition of states on the red line), and it cannot be zero in any topological edge state. The direction of the magnetic field sets the sign of the slope (which determines the direction of the edge current), and the strength of the magnetic field sets the size of the bandgap. Now, if disorder is introduced into this structure, the bands will be slightly modified, and the slope of the red line will be slightly altered. But as long as the strength of the disorder (random variation in the potential) is smaller than some value, scattering will not couple the edge states to bulk states. The implication is that any disorder weaker than (approximately) the bandgap will not cause scattering into the bulk or backscattering. This is the origin of topologically protected transport in the quantum Hall effect, and its key ingredient is the size of the topological

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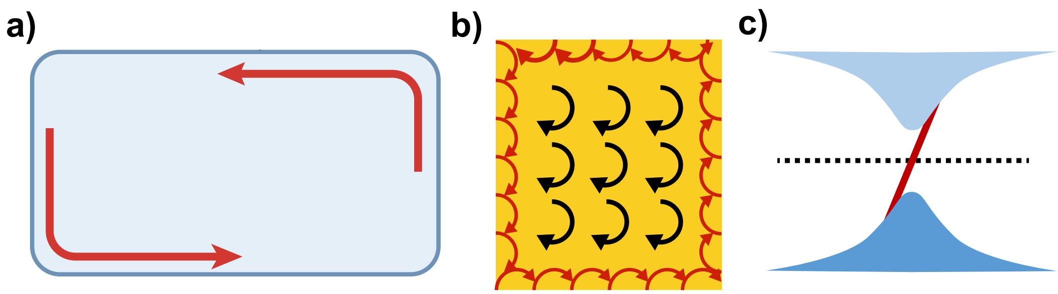

Figure 1: Topological insulators in a nut shell.
(a) Topological insulator: a two-dimensional (2D) material that is insulating in the bulk but exhibits perfect conduction on the edge.
(b) Simplified sketch of the integer quantum Hall effect, which was the first topological insulator. (c) Simplified dispersion relation for the quantum Hall effect, with the red line marking the topological edge states.

bandgap, which determines the degree of topological protection of transport. Modern topological insulators rely on the same principles, but the effects can be caused by a variety of other effects, among them fermionic spin–orbit coupling, external modulation, and crystal symmetries, etc. [3–6].

2 The road to topological photonics

Motivated by the growing success on electronic topological insulators, researchers started to ask whether the concept of topological insulators is unique to fermionic systems or is it actually universal. More specifically, can topological protection of transport exist in bosonic systems? Essentially – the challenge was to find a wave system whose dispersion curve resembles Figure 1c. The first suggestion was an EM system that requires breaking time-reversal symmetry to avoid backscattering [1]. Shortly thereafter, a more concrete idea was proposed [7] – based on gyro-optics materials where the application of a magnetic field indeed breaks time-reversal symmetry. This effect is fundamentally weak at all frequencies above THz; hence the resultant topological bandgap would be very small, providing essentially no protection of transport. However, at microwaves frequencies, the effect is strong, opens a large bandgap, and indeed within a year the EM analog of the integer quantum Hall effect was demonstrated [8]. At that point, the challenge was to find a new avenue to bring the concepts of topological insulators into photonics (optical frequencies and near infrared), without relying on the weak gyro-optic effects. Several ideas were proposed – ranging from using polarization as spin in photonic crystals [9] and aperiodic coupled resonators [10] to bianisotropic metamaterials [11]. Eventually, in 2013, the first photonic topological insulators were demonstrated [12], and it indeed displayed topological protection of transport (of light) against defects and disorder. That system relied on periodic modulation, which is the essence of Floquet topological insulators [6, 13]. In electromagnetism, employing temporal modulation in a spatially asymmetric system can lead to optical isolators that block backscattering [14], but still, relating modulation to photonic topological insulators required a lattice structure where the modulation can open a gap. As it turns out, a honeycomb lattice can play this role. Indeed, the first photonic Floquet topological insulators relied on a honeycomb lattice of coupled waveguides, where the modulation is generated by making the waveguides helical [12]. Around the same time, the aperiodic coupled resonator system was also realized in experiment [15] and demonstrated topological protection against disorder in the lattice [16]. Within a few years, numerous other EM topological systems were proposed and demonstrated, among them the topological bianisotropic metamaterials system [11, 17], the so-called "network model" of strongly coupled resonators [18, 19], and the crystalline topological insulator [20, 21]. These photonic topological systems are summarized in Figure 2. A recent comprehensive review on photonic topological insulators provides the details of these systems [23].

3 Photonic realizations of fundamental topological models

We are now more than a decade after the first demonstration of an EM topological insulator [8], and seven years after the first observation of the first photonic topological insulators [12, 15]. These experiments, and the pioneering theoretical papers during those first years, have created a new area from scratch: Topological Photonics. During

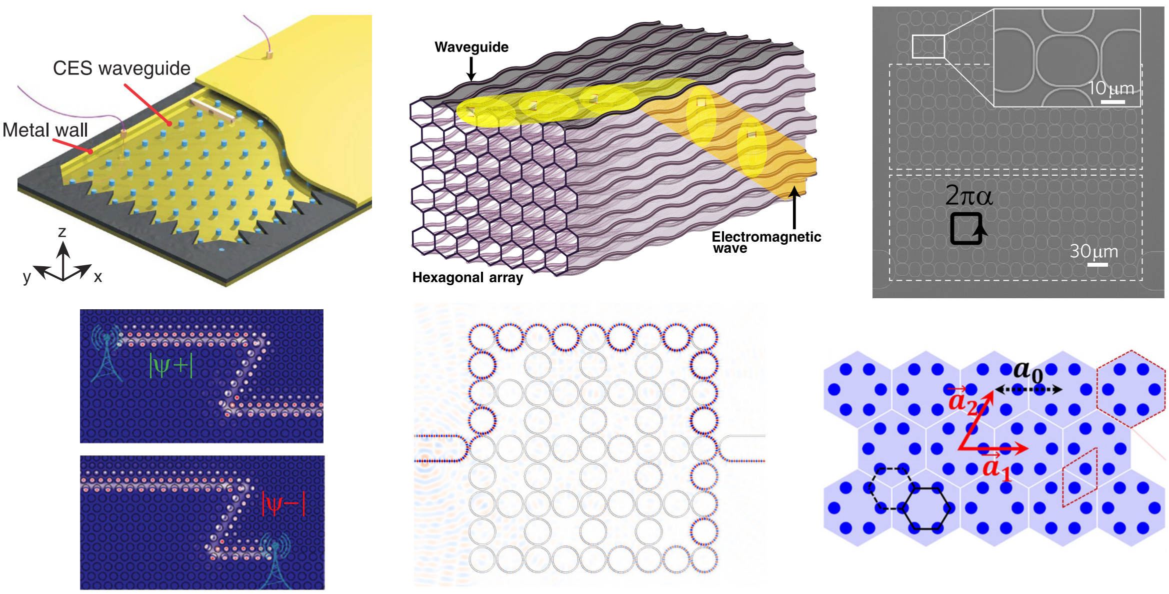

Figure 2: Various schemes for realizing topological insulators for electromagnetic (EM) waves.
Top, left to right: two-dimensional (2D) photonic crystal incorporating gyro-optic materials realizing the quantum Hall effect [8]; honeycomb lattice of helical waveguides realizing Floquet topological insulators [12, 22], aperiodic resonator array realizing the quantum Hall effect [10, 15]. Bottom, left to right: photonic topological insulator based on bianisotropic materials [11, 17], the network model of photonic topological insulators [18, 19], the crystalline photonic topological insulators [20, 21].

these years, this new area has gone a long way in multiple direction. One of the most important directions is ***using photonic systems as platform to experiment with fundamental concepts in physics***, which have been proposed but never realized in experiments. Namely – using photonic platforms to realize models that have been suggested in condensed matter physics. In fact, such photonic realizations were the starting point of topological photonics anyway: both the gyro-optic microwaves system [8] and the aperiodic coupled resonator systems [15] realize the quantum Hall effect, while the helical honeycomb lattice [12] realizes a Floquet topological insulators [6]. Likewise, the valley Hall photonic topological insulator [24] emulates the topological valley Hall transport in bilayer graphene [25]. Interestingly, sometimes the photonic realization of topological insulators came before the analogous experimental observations in solid state, for example, the first Floquet topological insulator ever realized in experiment was the photonic one [12], and it was followed by the solid state realization in Bi_2Se_3 [26]. Another example is the anomalous Floquet topological insulators, which were originally proposed in solid state [27], first observed in photonics [28, 29] and very recently observed with ultracold atoms [30]. In a similar vein, the topological Anderson insulator [31] – a fundamental system which becomes topological only through the introduction of disorder – was first demonstrated in photonics [32], and shortly thereafter with ultracold atoms [33], but realizing this idea in electronic condensed matter systems (in the context it was proposed) seems like a remote possibility. Undoubtedly, the realization of new topological systems, and generally of new phenomena that otherwise cannot be observed in the context they were proposed, has much value. Theoretical work and simulations always idealize the system and assume that the governing equations represent all the relevant physics involved, whereas experiments are never completely isolated from additional effects, and above all, the experiment itself often leads to new ideas and many times offers surprises.

Finally, and most importantly, photonics brought several fundamental and exclusive discoveries on topological physics, by combining non-Hermitian (NH) and topological physics as we describe below. Among those, the most important one (in terms of both fundamentals and applications) is the topological insulator laser [34, 35].

4 Topological photonics in tailored lattices

Another class of phenomena where topological photonics makes a big difference are those that were originally proposed in photonics but have later proved to be universal and could be observed in other fields beyond the domain of EM

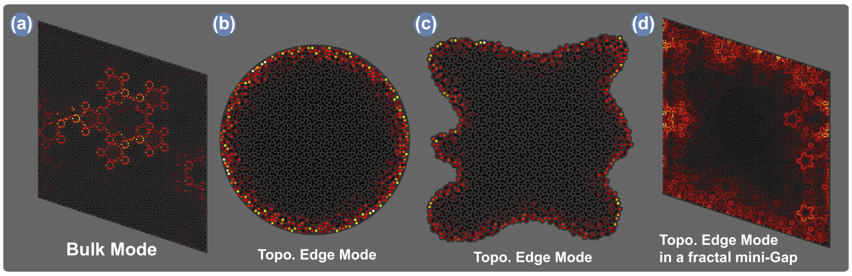

Figure 3: Two-dimensional photonic topological insulator quasicrystals [36]. (a) Typical bulk mode of a quasicrystal lattice. Topological edge states of periodically driven quasicrystals [36], shaped as (b) circle, (c) arbitrary shape, and (d) a topological edge state that lives in a fractal mini-gap.

waves. For example, quasicrystal topological insulators without magnetic fields [36], which were proposed in photonics but the underlying ideas are universal. This system is especially intriguing because the principles of topological insulators seem to require the existence of a bandgap, which is conceptually associated with a periodic system that has a well-defined unit cell, whereas a quasicrystal is not periodic and has no unit cell; rather, its spectrum is fractal. Nevertheless, under periodic modulation quasicrystals can be topological insulators, and its topological edge states exhibit all the features of topological edge states. Figure 3 shows the topological edge states of Floquet quasicrystals of different shapes, highlighting the fact that the shape of the edges is unimportant for having topologically protected transport. In a similar fashion, it was recently suggested that, under periodic modulation, even fractal structures can become topological insulators [37]. For example, the Sierpinski gasket, which has a Hausdorff dimension of 1.585, exhibits topologically protected edge transport even though it has no bulk at all: every site resides on an edge, external or internal. Nevertheless, the periodic modulation of this lattice has topological edge states on its exterior and on its internal edges.

Topological photonics can also offer new ideas that have no counterparts in other physical systems, such as the recent proposal [38] for broadband topological slow light through higher momentum-space winding, which can greatly enhance light–matter interactions.

5 Topological photonics in synthetic space

Another direction where photonics is having an important impact is topological physics in synthetic space. Despite different manifestations in many physical systems, topological insulators usually rely on spatial lattices. The wavepackets propagating in the lattice, whether electrons, photons or phonons, are subjected to gauge fields that give rise to the topological phenomena. However, lattices, periodic structures, do not necessarily have to be a spatial arrangement of sites. Rather, a lattice can also be a ladder of atomic states, or photonic cavity modes, or spin states. Using one (or more) of these ladders in a non-spatial – but synthetic – degree of freedoms, requires the introduction of coupling between the synthetic sites, which can be achieved by external perturbation. In contrast to traditional topological insulators based on a spatial lattice, for topological insulators in synthetic dimensions the transport is not restricted to the spatial edges of the system, but rather transport occurs on the edges of the synthetic space. In this way, it is possible to have topologically protected transport extending over the bulk in real-space. For example, the lowest and highest modes in a system serve as synthetic edges. Based on this concept, topological edge state were observed in cold atoms system, using atomic spin states as a synthetic [39, 40], or the atomic momentum states of a Bose–Einstein condensate [41]. However, using internal degrees of freedom for implementing the synthetic dimensions involves several fundamental problems: the number of these states is small, and the excited states always have a short lifetime. Here is where photonic comes into play and offers numerous ways to realize synthetic dimensions through equally spaced modes of the system. Synthetic dimensions in topological photonics were first introduced for topological pumping [42], where a photonic lattice was mapped onto a corresponding quantum Hall lattice with twice its spatial dimensions. In this vein, photonic topological insulators in synthetic dimensions were proposed with the synthetic space realized through cavity modes [43–45] which offers not only an unlimited number of states but also long lifetime, both marking big advantages for large-scale lattices. More recently, photonic topological insulators in synthetic dimensions were demonstrated in experiments [46]. That system (described in Figure 4) consisted of a two-dimensional (2D) waveguide array, engineered such that it is effectively a 2D lattice, where one of its dimensions is an ordinary spatial dimension, but the second dimension is the mode spectrum of each column of waveguides. This construction enabled observing of the dynamics of topological edge states in synthetic space, highlighting the topologically protected transport [46]. The beauty of this scheme is that it allows for

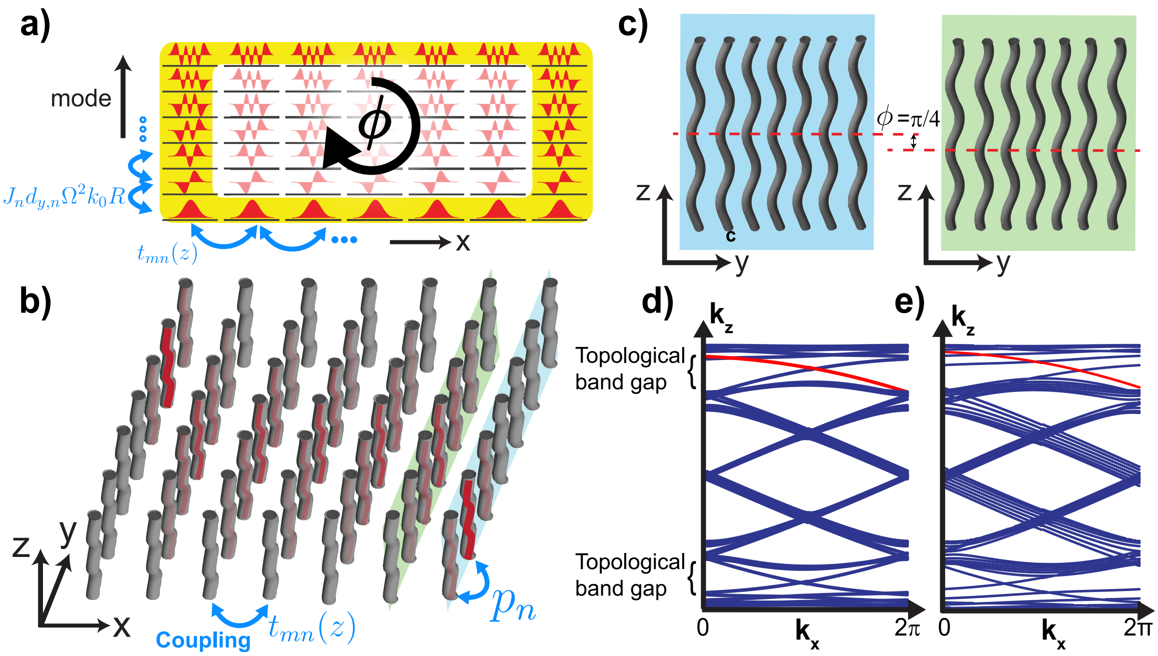

Figure 4: The two-dimensional (2D) synthetic space photonic topological insulator made of an array of judiciously modulated waveguide [46]. (a, b) The synthetic space lattice (a) corresponding to the two-dimensional lattice of waveguides in real-space (b). The basic building block is a one-dimensional (1D) array of *N* evanescently coupled waveguides, with the spacing between waveguides judiciously engineered such that it yields *N* Bloch modes with equally spaced propagation constants [49]. To facilitate transport in the model dimension, the modes are coupled by spatially oscillating the waveguides in the propagation direction, creating a ladder of coupled modes. Arranging *M* such 1D arrays next to one another, with the oscillations phase-shifted from one another, results in the 2D topological insulator in synthetic space. The edge state of the synthetic space (yellow in (a)) resides in the bulk of the waveguide array of (red in (b)). (c) The phase shift between each two adjacent columns of the waveguide array of (b). (d) The Floquet band structure of the lattice with the edge state marked by the red line. (e) The Floquet band structure under random disorder in the coupling between waveguides. Disorder mostly causes shifts and slight deformations in the dispersion curve of the edge state but does not close the topological gap, highlighting the immunity of the topological edge state to disorder.

increasing the dimensionality further to three-dimensional (3D) and even four-dimensional (see Supplementary Material of [46]). Indeed, a closely related system was very recently demonstrated to display topological transport in a 3D synthetic space [47]. In a different photonic realization, dynamics in two synthetic dimensions has been recently demonstrated in a scheme based on a single temporally modulated ring cavity [48]. In this realization, the synthetic dimensions were the frequencies of the cavity modes and the pseudospin states of the clockwise and anticlockwise states. This simple configuration, albeit physically consisting only of a single ring, facilitated demonstrating a variety of effects such as effective spin–orbit coupling, magnetic fields, spin-momentum locking, Meissner-to-vortex phase transition, and signatures of topological chiral one-way edge currents, all completely in synthetic dimensions. This paper demonstrates a new kind of topological protection: transport in synthetic space here means conversion from one mode to another mode, where the topological landscape guarantees robustness to the modal conversion process.

It is now already clear that utilizing the arsenal of photonics to create experimental schemes for topological physics in synthetic dimensions offers a plethora of new possibilities that can hardly be matched outside photonic. For example, it is possible to include additional frequencies to the oscillating 1D columns there, which would induce long-range coupling upon design – leading to new unexplored models of lattice geometries. Finally, adding gain and loss to such lattices in synthetic space would pave the way to party-time (PT) symmetry and exceptional points in synthetic space [45].

6 Topological quantum photonics

The topological protection of transport is in principle a wave phenomenon. However, in condensed matter it was argued that the topological robustness can also protect entanglement, by protecting the entanglement carriers against decoherence [50]. Unlike condensed matter systems where the issue of decoherence is a major obstacle for any application involving quantum information, in photonic systems there is no need to protect photons from decoherence because photons barely interact and decohere slowly. So what does it mean to protect photonic quantum information? The notion of "topological protection of entangled photon states" was first introduced in [51, 52], where it was shown that photonic topological insulators can be used to robustly transport fragile biphoton states. It was shown through simulations that these states maintain their path entanglement despite disorder, in stark contrast with non-topological systems. The topological robustness of entangled photon states is manifested in the robust transport of its unidirectional propagating edge states, where scattering by defects and imperfections are suppressed. Since in photonic quantum information, the scalability to large systems is limited by scattering loss and other errors arising from random fabrication imperfections, the hope is that topological architectures of the photonic circuitry will help in facilitating photonic quantum computing [53]. Proving that topological settings can give rise to robustness of multiphoton quantum states, specifically in architectures relevant to quantum computing, would hold great promise for fault-tolerant quantum logic, which is otherwise very fragile in large-scale settings such as quantum information systems.

Experimentally, transport of quantum edge states using single photons has been demonstrated [54–56], and it is of clear interest for quantum simulation and sensing. These single-photon experiments studied the physics of topologically protected bounds states [55] and topological transitions [56] in photonic quantum walks, as well as demonstrating an interface between a quantum emitter and a photonic topological edge state [54]. However, quantum information systems rely on multiphoton states. Recent experiments in a 1D binary array of waveguides have demonstrated topological protection of biphoton correlations [57] and of entangled photon states [58], which are the key building blocks for robust quantum information systems. These experiments showed that the biphoton states maintain their spatial distribution in the high-dimensional Hilbert space and their propagation constant – as they propagate through a topological nanophotonic lattice with deliberately introduced disorder. In a different experiment [59], the topological edge states of the aperiodic resonator array were used as a platform for generating correlated photon pairs by spontaneous four-wave mixing, and it was shown that they outperform their topologically trivial one-dimensional (1D) counterparts in terms of spectral robustness. Generally, the research on quantum light in topological photonic systems has just started. Its main aspect – studying the topological protection of multiphoton states – was thus far demonstrated only in a 1D setting [58]. It is yet unknown if it can really penetrate into real quantum information technology, such as the large-scale silicon photonics circuitry for quantum computing that is now being developed by PsiQuantum Corp. Or perhaps there are other interesting applications that can make use of the topological protection of quantum light, e.g., the topological quantum-limited traveling wave parametric amplifier that is naturally protected against internal losses and backscattering [60].

7 Non-Hermitian topological photonics

Perhaps the most important fundamental aspect of topological insulators, where photonics is having a profound aspect is ***topology in NH systems***. Traditionally, NH operators have been used in quantum mechanics to describe loss mechanisms, open systems, finite lifetime and dephasing, which would otherwise have to be described by coupling to degrees of freedom outside the system of interest. In this context, the NH version of quantum mechanics is helpful in simplifying calculations, identifying resonances, etc., but the underlying assumption was always that all the observables of a physical system must be real quantities, and consequently the operators must be Hermitian. Twenty years ago, it was found that NH Hamiltonians that obey PT symmetry have a regime of parameters where all their Eigenenergies are real [61]. That discovery implied that, possibly, NH operators can represent physically observable quantities. Still, for another decade that discovery remained with limited physical consequences, until Christodoulides, Makris, El-Ganainy and Musslimani [62], and shortly thereafter independently Moiseyev, Klaiman, and Gunther [63], introduced the concepts of PT-symmetry into optics. The first experiments followed within two years [64, 65]. Actually, introducing NH into optics is very natural, as the NH parts of the operators represent gain and loss, which are present in any laser system. Since then, the field of PT-optics, or in a broader sense – NH optics – has been overwhelmingly flourishing with research activity.

As an important part of the vision for topological photonics was to explore new universal concepts, combining topology with NH physics was a natural but highly challenging goal. Indeed, the first NH topological system was demonstrated in a photonics experiment [66], following an earlier prediction of a topological transition in NH quantum walk [67]. This was a 1D NH lattice, where the presence of loss made it possible to identify the topology of the corresponding passive NH system just from bulk measurements, without the need to investigate edge states. But irrespectively, the combination of topology and NH was bound to create controversies and arguments, at the very least – because topological physics relies on the existence of topological quantities that remain invariant during deformations of the system, and in NH systems it was not clear at all that such quantities can exist. In this spirit – there were theory papers that claimed, for example, that PT-symmetric topological systems cannot exist, casting major doubts on the ability of NH topological systems to exhibit topologically protected transport. Some of the controversy was resolved by the demonstration of 1D topological photonic systems exhibiting broken [68] and full [69] PT-symmetry. However, these 1D systems cannot have topologically protected transport along their edge because the edge in a 1D system is zero-dimensional (0D). The existence of NH topologically protected transport was highlighted by the recent discovery of topological insulator lasers [34, 35]. These are lasers whose cavities are specifically designed to support transport along the cavity edges, while making use of the topological immunity to defects and disorder to enhance the lasing efficiency and maintain single-mode lasing even high above the threshold [34, 35]. One of the models for topological insulator lasers is based on the Haldane model with the addition of gain, loss and nonlinearity [34], where it was shown, unequivocally, that the topological platform give rise to immune transport in this laser system, despite the non-Hermiticity. At that point – it became clear that topological invariants should exist also in NH systems, otherwise – there would be no explanation to the unidirectional transport and the immunity to defects and disorder [34]. This was the goal of several recent papers [70–75]: to present a general framework for classifying topological phases of generic NH systems. This fundamental issue is still one of the outstanding challenges of topological physics at large, where topological photonics is the spearhead of research.

8 Topological insulator laser

Undoubtedly, in the entire field of topological photonics, the research topic that is closest to real technological applications is the topological insulator laser. The vision here is to harness the features of topologically protected transport to force many semiconductor emitters to lock together and behave as a single powerful highly coherent laser source. Technologically, having a high power semiconductor laser has been a challenge of more than four decades, and all attempts to make a "broad-area laser" or a "laser diode array" have generally failed. Laser arrays are currently used only as a strong flashlight to pump solid state lasers (NdYag, etc.), but their coherence is not much better than of a light emitting diode. The vision was therefore to make use of the fundamental features of topological insulators to force injection-locking of many semiconductor laser emitters to act as a single coherent laser [76]. But in the way stood the question of whether NH systems (such as a laser) can support topological protection of any kind. Shortly thereafter, there was a series of works demonstrating lasers emitting from a 0D topological edge state in a 1D chain [77–80], but in those systems there is no edge transport at all, hence no protection to onsite disorder, and the lasing is almost fully confined to a single resonator. Then, there was an attempt to incorporate gyro-optic material in a laser cavity [81], but since the magneto-optic effects at optical frequencies are extremely weak, the bandwidth of the laser was broader than the topological bandgap. The first topological insulator laser was actually demonstrated in experiments a few months earlier [79] and it displayed all the expected features (see Figure 5) [35]. This laser was constructed on a standard optoelectronic platform, as an aperiodic array of 10 × 10 coupled ring-resonators on InGaAsP quantum wells wafer. This 2D setting is comprised a square lattice of ring resonators coupled to each other via auxiliary links. The intermediary links are judiciously spatially shifted to introduce a set of hopping phases, establishing a synthetic magnetic field that yields topological features. To promote lasing of the topologically protected edge modes, only the outer perimeter of the array was pumped, while leaving the interior elements lossy. This topological insulator laser operates in single mode, even considerably above threshold, whereas the corresponding topologically trivial realizations lase in an undesired multimode fashion, see Figure 5. More importantly, the topological laser displays a slope efficiency that is considerably higher than in the corresponding trivial realizations, even in the presence of defects and disorder [35].

Since that visionary work [34, 35] several groups followed with a variety of configurations for realizing topological insulator lasers, e.g., a topological quantum cascade laser with valley edge modes [82], topological bulk laser based on band-inversion-induced reflection [84], and a topological insulator laser with next-nearest-neighbor coupling [85]. Finally, we note very recent experiments on a topological vertical cavity surface emitting laser [86]. Importantly, very recent theoretical work [83] showed that indeed the topological design greatly improves the coherence of a large array of emitters, as envisioned by [34, 76]. From all of this activity, it is now quite clear that currently the topological insulator laser is the most promising application of topological photonics, with many new ideas

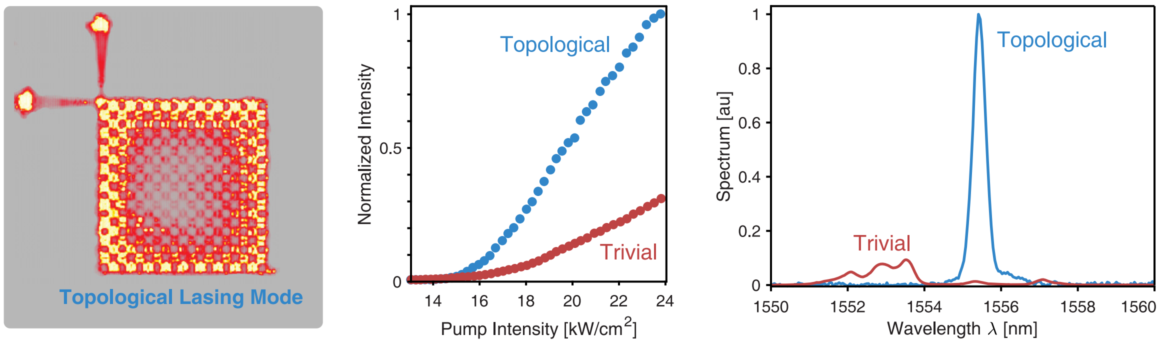

Figure 5: Topological insulator laser [34, 35].
Left to right: Top view photograph of the lasing pattern (topological edge mode) in a 10×10 array of topologically connected resonators, and the output ports. Output intensity versus pump intensity for a topological insulator laser and its corresponding trivial counterpart. The enhancement of the slope efficiency is approximately threefold. Emission spectra from a topological insulator laser and its topologically trivial counterpart.

emerging, for example, utilizing topology in synthetic dimensions to force an array of semiconductor lasers to emit mode-locked pulses [87], which could overcome a challenge of three decades.

9 Conclusions

We attempted to provide our perspective on the new field of topological photonics, which is currently at the forefront of photonics research and is also the spearhead topological physics at large. We covered here a small selected list of topics, but in fact there are many more. For example, we did not cover nonlinear topological photonic systems [88], which have started to attract much attention recently with the observation of solitons in a topological bandgap [89]. Likewise, we did not discuss topological exciton–polariton settings [77] and exciton–polariton topological insulators [90], which are extremely interesting because they are a topological symbiosis between light and matter. We did not discuss a plethora of many new ideas that are now frequently emerging in topological photonics. Altogether, it is now clear that within the past decade this new field has gone a long way, and it continues to generate new fundamental ideas and offer exciting applications. Can some of these topological applications become real technology? We can carefully say that the answer seems to be positive, but we will know a more definite answer within five years.

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